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Rational Procedures[1]
A neo Fregean Perspective on Thought and Judgment

Summary
In this paper I shall deal with the role of "understanding a thought" in the debate on the definition of the content of an assertion. I shall present a well known tension in Frege's writings, between a cognitive and semantic notion of sense. This tension is at the source of some of the major contemporary discussions, mainly because of the negative influence of Wittgenstein's *Tractatus*, which did not give in-depth consideration to the tension found in Frege. However many contemporary authors, after the first attempt by Carnap himself, have tried to make room both for a cognitive and semantic aspect of meaning. I claim that at least some of these attempts (Dummett, Perry and Chalmers) are seriously flawed, mainly due to the difficulty in making a proper connection between the two different conceptions of sense. I shall outline an alternative project, which takes into consideration Frege's requirements on antipsychologism and of the objectivity of thought, while maintaining a close connection between the two aspects of sense. The hints which may come from Frege's tension suggest some constructivist solutions, solving the tension within a framework of contemporary logic of uncertain reasoning.

1. Understanding and Judgment

Frege distinguished between understanding and judgment. Understanding is a mental *process*, the most mysterious of all because it links the subjective realm of the consciousness to the objective realm of thought. Judgment is a mental *act*, the act of recognizing the truth of a thought; this act finds its proper expression in the speech act of the assertion.

In recent discussions on Frege, many authors, among whom Brandom 1994, have insisted on the original definition of sense given in *Begriffsschrift* §3: the sense of a sentence is its inferential potential. Therefore to understand a thought means grasping the consequences of what is said, while asserting a sentence entails endorsing a commitment to the set of the consequences closed inside the conditional relation (and also a commitment to answer for the justifications which entitle one to assert the sentence).

While an assertion is something dealing with a unavoidable normative aspect, a discussion of understanding means entering a descriptive task: how does understanding work? How can we describe this mental process? On the face of it, Frege oscillates between two different attitudes in discussing understanding: on the one hand he claimed to give a theory of "the mind", that is an abstract theory of the ideal mind and not of the actual minds. On the other hand, he gave examples of limited understanding and was well aware of the limitations of human minds in understanding thoughts.

In this paper I shall deal with the problem of treating the topic of limited understanding seriously, using some tenets of Frege's writings as suggestions for our present worries not only in formalizing mathematical reasoning, but also commonsense reasoning.

2. Two conceptions of sense

Since Carnap 1947, it has been widely recognized that Frege's notion of sense is not a clearly defined and unitary one. Some authors, like Kripke and Putnam, have tried to show that some features of the notion of sense (determining the reference and being mental content) cannot hold together. Other authors insist on the duplicity of the notion of sense, claiming that there are at least two basic notions

of sense, labelling them with different terminology: we have, then, on the one hand a semantic, logical, ontological view, and on the other a cognitive or linguistic one[2]. Recently Dummett also came to accept the idea of two different notions of sense in Frege[3]. I shall speak here of the tension between a "cognitive" notion and a "semantic" notion of sense.

One of the most well known definitions of sense given by Frege is that the sense of a sentence is the thought expressed by a sentence. In a well known passage Frege claims that sentences with the same truth value may express different senses: *Hesperus is a planet* and *Phosphorus is a planet* express different senses because a rational person may believe one true while being agnostic as to the other. This suggestion has been called the "intuitive principle of difference of thoughts" (Evans 1982) or "principle of rational cotenability" (Sainsbury 1999). Few people have noticed that Frege used a stronger version of the principle, which implies that also logically equivalent sentences may express different senses or thoughts. In his letters to Russell and Peano he provides many examples of mathematical equations, and this idea is also given in his *Grundgesetze*: "2+2=4 and 2×2=4 express *different* senses" (*Frege* 1893, §). Given his logicist convictions, this claim should be read as if instead of two mathematical formulae we had two logically equivalent logical formulas. In fact, if sense is compositional, the two sentences have different components (the function "+" and the function "×"), and therefore express different senses. The same should also happen with other pairs of logical equivalent sentences such as "if p then q" and "not (p and not q)".

This conclusion contrasts with a strong assertion made by Frege in a letter to Husserl about what we should consider the sense of, or the thought expressed by, two logically equivalent sentences. He claims that two logically equivalent sentences, like "if p then q" and "not (p and not q)" express the *same* thought (*LH* 1906). Here we have an apparent tension in his treatment of the concept of sense.

3. Fregean motivations for the cognitive and semantic notions of sense

The motivations for a "wide" or semantic conception of sense are straightforward, and appear to be linked to two kinds of criteria of identity of senses (i) logical equivalence (ii) immediate recognizability[4]:

(i) in a letter to Husserl in 1906, Frege on the two above mentioned sentences ("if p then q" and "not (p and not q)") says something like "Just take my *Begriffsschrift* and you will see that they express the same thought. To look for some more detailed relation of congruence will bring us to endless and useless discussions."[5] Here he seems to be saying that there is no conception of sense identity which specifies any criteria stricter than logical equivalence or identity of truth conditions.

(ii) Frege also says[6] that two sentences A and B have the same content or are equivalent when anyone who recognizes the content of A as true must immediately also recognize that of B as being true, and vice versa. This criterion is based on the assumption that there is no difficulty in grasping the content of A or B. In his 1923 paper, discussing compound thoughts, Frege appears to use the criterion of immediate recognition to justify the sameness of sense of logically equivalent sentences: that "*B and A*" has the same sense as "*A and B*" can be seen, without the need of proof, as soon as

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[2] See for instance Currie 1985, Beaney 1994, Garavaso 1991,Bell 1996, Penco 2003. Künne 2007 distinguishes between "expressing" a thought and "articulating" it.

[3] Referring to the difference between "Cato killed Cato" and "Cato killed himself", Dummett 1997 (2005 p. 159) says "We surely need to distinguish two senses of "sense", one in which they do have the same sense and the other in which they do not." The difference here is given by the procedure (meaning, character…) connected with the reflexive pronouns, therefore linked to indexicals and context of utterance. Dummett tends to think that, setting aside matters of context of utterance, we should keep a laxer sense of "sense" in which the two sentences have the same sense. I suggest that the problem is not only connected with the context of utterance, but extends more generally.

[4] The discussion on criteria of identity of sense is wide-ranging. See Penco 2003 for a comparison of different criteria of identity discussed by Frege.

[5] This is not a literal quotation, but a summary of what Husserl says at pp.103-105 of WB.

[6] The best presentation of this criterion is given in *A Brief Survey of my Logical Doctrine* (1906, NS p.213).

we grasp such a sense.

On the other hand, there are strong motivations for the cognitive notion of sense based on the intuitive criterion of *difference* of senses. Let us see at least two cases which show deep motives for which Frege should hold sentences like "if p then q" and "not (p and not q)" to have different senses[7].

(i) The intuitive Criterion of difference of thought or the Rational Cotenability criterion suggests that a rational person can make mistakes not only through ignorance of empirical matters, but also through ignorance of logical procedures or miscalculation, and still be rational. The fact that expressions such as 7=7 and $5^2 \times 211-4/753 = 7$ are considered as having different senses is an example[8] of this attitude: somebody might mistakenly think that the second sentence is false while obviously believing the first to be true. Here is a case where the Intuitive Criteria of differences of thought pertains to computational difficulty instead of empirical ignorance (even if we may also think of the case of "Hesperus is a planet" and "Phosphorus is a planet" as a case of computational difficulty in astronomical calcula)[9].

(ii) We learn something new when we learn a new way to formulate a thought; "Do we learn anything new upon hearing it [if p then q]? Well, before hearing this sentence someone may not have known and hence not have *acknowledged this truth* at all. To that extent one can, under certain circumstances, learn something new upon hearing it".[10] The case of learning is explained by the different composition of thoughts; if two thoughts have different components, and each component has its peculiar sense, then the two thoughts with different components should have different senses. The clearest example of this attitude is given in Frege's paper on negation, written in the same period as his paper on compound thoughts; Frege asserts that A and its double negation not-not A have different senses (although it is clear that they are immediately recognizable and have the same truth conditions)[11].

A passage in a letter to Husserl in 1906 helps explain why Frege found it difficult to keep the balance between these two attitudes; on the one hand he seems to require two levels of sense, on the other hand he puts anything cognitive in the realm of psychology: "We may have a unique normal form (expressing the thought), and different expressions for different purposes, to be used in different circumstances for psychological reasons"[12]. This attitude is reminiscent of his idea of *Begrifsschrift* 3, where Frege says that different sentences are equivalent from a logical point of view when they lead to the same consequences, although you may find some "small difference in sense" among them, which are relevant for pragmatic reasons. Pragmatic reasons deal with the understanding of speakers, hence with the subjective realm, and Frege avoids the intrusion of psychology in logic. However the use of different expressions useful for different purposes also reveals something objective. The motivation to use one expression rather than another may be psychological, but the different kinds of expressions may be studied objectively. They may reveal different kinds of computation, therefore something objective which has nothing to do with psychologism.

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[7] A more detailed analysis of using the Intuitive Criterion of difference of sense also for logical equivalences is provided in Penco 2003a, where there is a short consideration on the different alternatives in interpreting Frege on this tension (Did he change his mind? Did he make any mistake?)

[8] The example is given in a letter to Russell (November 1904 WB p. 247) commenting that cognitive value does not depend only on *Bedeutung*, but also on *Sinn*; other similar examples with the same point are found in letters with Peano where, besides the Hesperus-Phosphorus example, Frege gives mathematical examples like "4+3>>2+3" as having a different sense from "5+2>5" (correcting some misprint of Frege's or the editor's).

[9] It might be objected that we cannot use the criterion of rational cotenability in cases where we entertain different attitudes towards two logical equivalent sentences. However, given our limited understanding – which Frege recognises as such – what we must be content with is the rationality given by a method to verify and recognize mistakes. The empirical case is not so different from the logical case. We are rational if we think it true that Hesperus is a planet and false that Phosphorous is a planet; but we are ready to change our minds when exposed to new information on the identity of the two celesital bodies. In the same vein, we are rational if, understnding the meaning of the logical constants, we give – by mistake, distraction or default – two different semantic evaluations to two logically equivalent sentences, but we are ready to change our judgements when exposed to some convincing proof. I have treated this aspect in more detail in Penco 2003a.

[10] See the discussion given in Künne 2008, who quotes this passage.

[11] NS 156-157.

[12] Letter to Husserl 1906: WB p. 102

4. *Tractatus'* restrictions and Carnap's reaction

Even if, in *Grundgesetze der Arithmetic* § 32,[13] Frege hinted at the idea of the sense of a sentence as truth conditions, Wittgenstein deserves the merit for making the concept clear and unambiguous. The sense of a sentence is given by its truth conditions, and understanding the sense of a sentence is understanding the conditions under which the sentence is true[14]. However Wittgenstein did not accept Frege's oscillations, making most philosophers think of Frege's conception of thought as a straightforward identification with truth conditions. Künne 2008 devoted some time to showing the rigid attitude of the early Wittgenstein towards the wider and more sophisticated assessment held by Frege; actually Wittgenstein cut the Fregean connection between understanding and sense because he could not accept any worry of the kind Frege had; it would have meant falling into epistemological matters, and – given that epistemology is the philosophy of psychology – he did not want to run the risk of "[getting] entangled in unessential psychological investigations".[15]

But Wittgenstein's *Tractatus* wasn't the only heir of the Fregean conception of sense. Carnap, as a pupil of Frege's lectures at Jena, has the right to be considered as important as Wittgenstein in developing Frege's ideas; like Frege he distinguished sharply between cognitive meaning and emotive meaning, and he attempted to give a new logical theory, treating the same problems Frege wanted to treat, like compositionality and substitutivity. Actually he did not accept Frege's distinction between sense and reference and developed a more traditional distinction between intension and extension, giving a formal explicatum of the traditional notion of intension along the lines of the concept of Wittgensteinian truth conditions. However, differently from Wittgenstein, Carnap thought that the notion of intension (truth conditions) is not enough to treat belief contexts. We need some other conceptual tool to deal with the cognitive limitations of speakers, because - taking the above mentioned Fregean example - somebody may believe that 7=7 and disbelieve $5^2 \times 214/753=7$. Carnap, the heir of logicism after Frege, would say that the two expressions, although logically equivalent, or having the same intension, have different intensional structures, given that the form of "$5^2 \times 214/753=7$" is much more complex than "7=7". In order to keep substitutivity in belief contexts Carnap required intensional isomorphism, i.e. identity of intensional structure, where two sentences have intensional structure if they "are *built in the same* way out of corresponding designators with the same intensions" (Carnap, 1947 §14).

Unfortunately Carnap referred to intention as an explication of the "cognitive meaning" contrasted with the emotive meaning; in doing so he did not help his own claims greatly, and many authors still refer to his concept of intension as the main concept expressing cognitive significance (e.g. Chalmers 2006). On the other hand, it is the idea of intensional structure and intensional isomorphism – not the idea of intension – which contributes the most interesting attempt at vindicating Frege's conception of the cognitive aspect of sense. However the idea of intensional structure has a disadvantage; even if it is not reducible to a syntactic notion – as Carnap points out – it is too restricted to the form of the component expressions. Therefore, while it works for cases like the mathematical case above, it does not work to explain the failure of substitutivity in other simple cases of belief contexts. For instance it makes "Hesperus is a planet" and "Phosphorus is a planet" intensionally isomorphic, contrary to the idea that they express different thoughts or senses. Even though it was a serious attempt to keep the attention on a real cognitive problem, Carnap's solution has not been very influential in the development of logic and theories of meaning.

With the failure of Carnap's notion of intensional structure, semantic theories have been developed along the attitude of logical purity in the *Tractatus*. It is difficult to overstate the influence of Wittgenstein's *Tractatus* on contemporary conceptions of the content of an assertion. Model theoretic

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[13] Frege 1893, § 32, says that with our stipulations we determine in which conditions a sentence refers to the truth: the sense of a sentence is therefore defined as "the thought that those conditions are satisfied".
[14] Wittgenstein 1921 § 4.431 however assumes that Frege had already given truth conditions as an explanation of the signs of his *Begriffsschrift.*
[15] See Wittgenstein 1921, 4.1121.

semantics after Kripke's possible world semantics has developed the conception intension with almost no space left for other cognitive aspects, confining the discussion on the topic of conceptual or inferential role semantics. Discussion on the tension between the two Fregean conceptions of sense has been mainly confined to the field of "Fregean studies" and has not provided any result outside the exegetical, although very interesting, discussion on Frege's ideas[16].

However, the abandonment of Fregean intuitions behind his cognitive conception of the sense problem has become a problem for model theoretic semantics and for the philosophy of language. Different attempts to ease the tension between the two contrasting Fregean intuitions have been proposed, often as an attempt to compose Kripkean intuitions and Fregean intuitions. From among the many proposals I shall select some which seem particularly suited to filling the gap (i.e. the ones I like most): Dummett's distinction between assertoric content and ingredient sense; Perry's distinction between semantic content and cognitive role; and Chalmer's distinction between subjunctive and epistemic intensions. These three are an exemplification of three trends in semantic theories: justificationist, dual aspect; and bidimensional. I shall briefly present these reactions to the "pure" account of the *Tractatus* and – after that – I shall make some short remarks on the possible flaws and limitations of these solutions.

5. Contemporary attempts to recover Fregean worries

Assertional content and ingredient sense. Michael Dummett tries the almost impossible task of mixing Frege's and Wittgenstein's ideas supplemented with intuitionistic logic. His basic idea is that we cannot be satisfied with the definition of meaning as a truth condition, but meaning has to be given in terms of a justification condition. Given the link between meaning and truth, this attitude has profound consequences on the concept of truth: a statement is true if it is (or was) possible to find a justification of it.

However, both justificationist and truth theoretical theories face a problem: if the meaning of a sentence is given in terms of truth or justification conditions, we could not explain the difference between sentences[17] like "it is sunny here" vs "it is sunny where I am". Certainly they have the same truth or justification condition, but – when inserted in wider sentences like "it is always sunny here" and "it is always sunny where I am" – they contribute differently to the meaning of the two sentences in which they appear. This apparently contrasts with the requirement of compositionality: if the two original sentences have the same content once evaluated in the context of the utterance, but change it when inserted in a wider sentence, then the two sentences do not contribute with their content to the meaning of the composed sentences in which they are embedded.

To overcome this difficulty Dummett suggests a distinction between *assertoric content* and *ingredient sense*. Following this distinction the two sentences like "it is sunny here" vs "it is sunny where I am" have the same assertoric content, but they have different "ingredient senses". This answer amounts to saying that there is *more* than assertoric content in an assertion – and what is more it is precisely the ingredient sense which seems to be the something similar to the cognitive aspect of sense we have been discussing above. The difference in this particular case is due, according to Dummett, to the fact that the adverb "here" is temporally rigid, while the phrase "where I am" is temporally flexible[18]. The ingredient meaning of the two sentences is therefore different, even if their assertoric content is the same.

- *Semantic content and cognitive role*: John Perry, on the basis of his interpretation of Frege, claims a sharp distinction between (truth conditional) content and cognitive role. "He is attacked by a bear" and "I am attacked by a bear" may have the same truth conditional content, but they have

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[16] See the papers mentioned in footnote 2.
[17] The example appears in different papers; see for instance Dummett 2004
[18] I would add that "where I am" can be interpreted both ways; I can say "I might not be here", but usually I cannot say "I might not be where I am", *unless* "where I am" is interpreted as a rigidifying description. The phrase "Where I am" has therefore an informative content richer than "here" and supports more interpretations.

different cognitive roles. In the first case I duck under the table trying to hide myself, in the second I run away. The cognitive role, given by the different linguistic tools used ("he" or "I"), has relevant consequences relating to differences in beliefs and actions. Same content, different roles: same truth conditions, different beliefs and different behaviour. The original distinction[19] is very similar to Kaplan's between content and character (or linguistic meaning). In Kaplan the character (or linguistic meaning) of an indexical is a way to give the content once given the context. In a Fregean way, cognitive sense would amount to something like character+context, where the context is part of the cognitive sense.

Differently from Kaplan, Perry develops a conception of different kinds or levels of truth conditions. A nice example is the one of Mach looking at himself in a mirror and, without recognizing himself, saying "that man is a shabby pedagogue". After a while he realizes that the man is himself, and thinks "I am a shabby pedagogue". Perry suggests defining different truth conditions in an "incremental" way[20]:

(i) "That man is a shabby pedagogue" is true if the person referred to by "that man" is a shabby pedagogue.

(ii) "I am a shabby pedagogue" is true if the speaker in the context is a shabby pedagogue.

(iii) "Mach is a shabby pedagogue is true if the person identified with the proper name "Mach" is a shabby pedagogue.

The three assertions contain singular expressions which have the same referential content, and different indexical content and designational content. The flow of information passes from perceptions used in temporary folders to enrich the information in higher level folders where beliefs about a person (beliefs about Mach) are placed. The connection between perceptions and ideas on Mach is fulfilled when the speaker recognizes that the person he sees is himself. We therefore have to recognize different levels of content, systematically correlated with one another. We have therefore an "incremental" theory of content, where the content of an assertion is evaluated with a truth value, always dependent on the level of content we are considering. Reflexive content is the level on which beliefs acquire the function to motivate actions. We keep the idea of meaning as truth conditions (and direct reference), but we may make room for the explicit recognition of different cognitive aspects.

- *Subjunctive intension vs Epistemic intension:* David Chalmers, developing some ideas of Robert Stalnaker, has become one of the main supporters of bidimensional semantics, one of the main formal tools purporting to give a unifying theory in which both semantic and cognitive aspects are treated together. The basic idea is that the extension (truth value) of an expression depends in two different ways on possible states of the world: (i) it may depend on the *counterfactual* world in which the expression is evaluated or (ii) it may depend on the *actual* world in which the expression is uttered. Therefore expressions have two kinds of intensions, which should capture two dimensions of meaning: (i) *subjunctive intensions*, which are the traditional functions from possible worlds to extensions, and (ii) *epistemic intensions*, which are functions from scenarios ("centred" possible words, characterized by a point of view and a time) to extensions. "Epistemic" intensions should account for the cognitive aspect of the Fregean concept of sense. The idea of scenarios or centred possible worlds depends on the idea of epistemic possibility: it is epistemically possible that *p* when the speaker cannot know *a priori* that *p* is not the case. Therefore "Hesperus is not Phosporus" is epistemically possible for a speaker ignorant of the identity Hesperus = Phosporus[21].

Chalmers remarks that epistemic intensions, though they may be given through descriptions, are not descriptions, but functions. In a possible world centred in a speaker's point of view, the epistemic intension of "Hesperus is Phosphorus" may be a function from possible worlds to the false,

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[19] I refer here to the distinction between "content" and "role", which will be superseded by the distinction between "referential content" and "reflexive content".

[20] see Perry 2001 §5.4 and 6.7; see also Perry's Genoa Lectures (Perry 2000)

[21] See Chalmers 2006. Chalmers' point is that it is epistemically possible that a centred scenario (e. g. where Hesperus = Neptune and Phosphorus = Mars, or where water is not H20) is actual. On the connections between Fregean sense and epistemic intension see also Chalmers 2002.

while the subjunctive intension of "Hesperus is Phosphorus" is a function from all possible worlds (in which Venus exists) to the truth. With this solution Chalmers seems to give a good rendering of Frege's sense as cognitive value (coherently with the criterion of intuitive difference of thoughts), without being compelled to identify senses with descriptions.

5. Disadvantages of recent attempts to recover Frege's duality of senses

- *Justification semantic*: The problem with the distinctions between assertoric content and ingredient sense is twofold. On the one hand, the ingredient sense is required to fill a gap in the justificationist theory of meaning, but its explanation seems circular: ingredient sense is what justifies the difference between assertoric content and ingredient sense. There is no specific and independent explanation of the ingredient sense. However, even if we accept the distinction as an enrichment of the assertoric or justificationist theory of meaning, it seems that, with the distinction being embedded in this type of theory of meaning, we should "buy" an antirealistic reconstruction of the notion of truth; but not everybody is disposed to take this step, especially if we want to preserve more of the original Fregean classical setting. Dummett's distinction, without its original motivations, runs the risk of being a free-spinning wheel.

- *Dual aspect semantics*: In traditional dual aspect semantics, narrow and wide contents become completely independent and perform different roles unrelated to each other (the mental domain and the social-causal domain). The main problem with these kinds of distinctions is therefore the strict separation of the two kinds of evaluations, one at a semantic level and the other at a psychological level; the first pertains to the objective or metaphysical realm of how things are; the other pertains to the mental domain, with the holistic aspect linked to our idiosyncratic set of beliefs. If we consider more strictly the development of Perry's way out we find another worry. We have a proliferation of different kinds of truth conditions which seem to go one after the other, without a real connection among them, so that they cannot always be taken together (in which case they would give contrasting interpretations). Even if Perry's attempt seems really to be a composition of Kripkean and Fregean worries, with its ambiguities it makes Frege's theory of sense completely "inexplicable"[22].

- *Bidimensional semantics*: Generally speaking, epistemic intension recovers only part of the Fregean cognitive notion of sense. We may re-describe the difference between "Hesperus is a planet" and "Phosphorus is a planet" saying that the two sentences may have two different epistemic intensions depending on different scenarios. However we need something more "cognitive" about this difference. And Chalmers recognizes that his notion of epistemic intension is not as fine grained as Fregean senses (Chalmers 2006 §4.1). Besides, there is a high price to pay for recovering part of Fregean intuition on cognitive aspects: the two kinds of intensions are not really properly connected; they are independent of one another; if not it would be difficult to avoid contradictory assignments to the same sentence in the same context (Hesperus is a planet from the point of view intension and is not a planet from the point of view of epistemic intension). If this is true, rather than being a unifying theory of meaning, bidimensionalism proposes two theories, each dealing with a different aspect or problem[23].

Bidimensionalism therefore appears to be not so distant from many previous attempts to deal with the original Fregean ambiguity. Actually most attempts amount to different answers to distinct problems: on the one hand a metaphysical or semantical problem, which leads to treating the meaning of a sentence as its truth conditions, and on the other hand an epistemological or linguistic problem, which leads to treating the meaning of a sentence as its cognitive import, relative to a

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[22] On the inexplicability of Frege's theory of sense see Perry 2001, p. 194.
[23] See on this aspect Marconi 2005. However inside Bidimensionalism we find different trends which deal with cognitive aspects in a different way; for instance Carpintero 2005 rejects any fixed distinction between the epistemic understanding of two dimensionalism (devoted to capturing *epistemic dependence*) and the contextual understanding (devoted to capturing *contextual dependence*). On the contrary, *any* distinction between a priori and a posteriori must be grounded in some context, and should pick out the personal-level semantic competence of a speaker. In this way we may account for differences in cognitive value between sentences like "Hesperus is a Planet" and "Phosphorous is a Planet".

speaker and a context. "Classical" truth conditions pertain to ontology or metaphysics, and application conditions, or the cognitive side of the matter, pertain to psychology or sociology or epistemology[24]. This is not so far from the judgment once given by Barbara Hall Partee: let us acknowledge that we have two different kinds of semantics, one dealing with mathematics and objective truth and the other dealing with psychology and mental matters. This attitude however would imply the failure of a unified treatment of Kripkean and Fregean worries and the acceptance of the most naïf aspects of Fregean antipsychologism. I think we should look for some more coherent and unitary framework of meaning and understanding, going beyond Frege's simplified antispychologism, without becoming "psychologized Fregeans"[25].

6. Cognitive Senses as *Rational* Procedures and three-layer semantics

Rethinking the original Fregean tension we may see a source of some of the difficulties in contemporary debate; and trying to find a way out of the Fregean tension may help to recover a fresh perspective on the relation between metaphysical worries and epistemological worries. A very intuitive way to show the compatibility of the two different Fregan conceptions of sense can be given by reasoning on two ways of interpreting the example discussed above in § 3, as a pair of sentences with the same semantic sense and different cognitive sense:

p q	If p then q	not	(p and not q)	
T T	**T**	**T**	F	F
T F	**F**	**F**	T	T
F T	**T**	**T**	F	F
F F	**T**	**T**	F	T

The truth tables show that the two sentences have the same truth conditions. Therefore – from the *Tractatus* point of view – they have the same sense. Certainly Frege thought something similar when he said that they express the same thought. However it is easy to see that the procedures for arriving at the same result are different; in the case of the conditional, we have one-step definition; in the other case we have negation and conjunction, and there are three steps to arriving at the result, showing a higher degree of complexity. Making it simple we may say that the two sentences express the same truth condition which is arrived at through two different computational procedures[26].

These different procedures can be considered as different ways of giving us the truth condition - the cognitive role of procedures is apparently enforced by more complex sentences, where it is easier to give different semantic evaluations because of possible mistakes, or because of miscalculations. As Frege remarks "we do not have the mental capacity to hold before our minds a very complex logical structure so that it is equally clear to us in every detail"[27] This and other passages drive us to recognizing that Frege was well aware that understanding is not a yes/no process; we need to recognize different levels of understanding[28].

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[24] Most interpreters use the ontology/epistemology distinction to separate the two notions: from an *ontological* point of view we have abstract thoughts, given by truth conditions and not by difference of linguistic expressions; from the *epistemological* point of view we may access thoughts only through language, and different sentences which may express the same thought, may express however different meanings (Bell 1996). Most scholars however agree that the tension in Frege cannot be properly solved without with some reconstruction. Frege used the term "sense of a sentence" ambiguously for different purposes in different contexts of his work (see Penco 2003a).

[25] I refer to the psychologized Fregean invented by Taylor 1995, defined by (i) something like Evan's principle (ii) semantic discernability is a condition of cognitive discernability (iii) theory of reference must provide routes of epistemic access to referents. Even if we accept the idea of fine grained procedures graspable by individuals, our perspective would reject tenet (ii), given the possibility of two indiscernible semantic sentences which are cognitively discernable.

[26] Other refined options on proofs as degrees of computational complexity are given in Penco- Porello 2007.
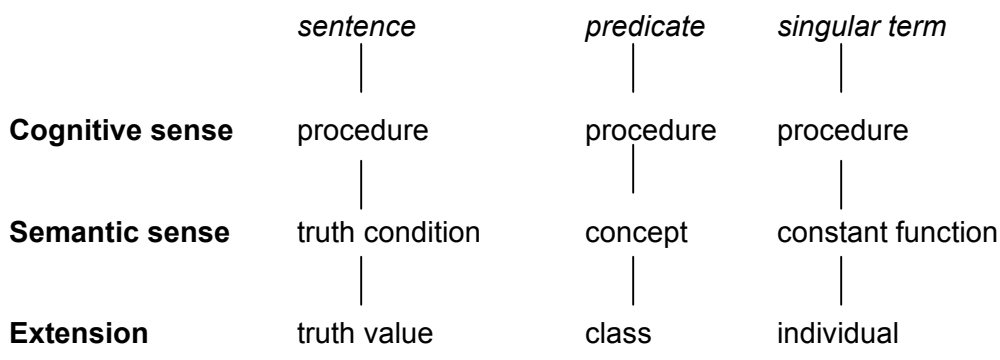
[27] See Frege, NS p. 240 ("Logik in der Mathematik", 1914).

[28] A way to do this is exemplified in Penco-Porello 2007. This aspect is already implied in the idea of sense as inferential potential; nobody can understand at a glance *all* the possible inferences which we may derive from our premises. However,

In evaluating sentences we have to rely on our *limited* understanding and consider not only the semantic sense but also the procedures we use to get at it. We should allow our formalism to give space to the recognition of our limited cognitive powers: if somebody holds "Hesperus is not a planet" as true, it is because of his lack of information, and we may represent the *reasons* for this lack of information, looking at the procedures he uses to individuate the object (either a perceptual procedure to see the object or a computational procedure to calculate its position in the sky). This way of facing problems is very different from the idealized way of bidimensionalism which imposes a principle of scrutability of reference and complete information[29]. I think we should also look for attempts in logic devoted to formalising uncertain reasoning, reasoning in the absence of complete information, or with inadequate computational capacities, or even with mistakes. The attention given by Frege to our limitation in understanding is only a first step, which was not pursued by Frege, but shows a direction of thought on how to treat bounded rationality.

Using contemporary terminology we might express the contrast in terms of intensions (functions from possible worlds to extensions) and procedures attached to intensions. In this setting the cognitive sense is represented by the procedures and the semantic sense is given by intensions. The cognitive sense intended as the procedure attached to a function constitutes the objective way in which the referent is given to us. However our limited capacities make us unable to always get everything right (for instance to get the right causal or anaphoric chain) and we may not recognize or perform completely or correctly the procedure we *should* perform. We may assume by default that we do it right, making room for the possibility to change our evaluations in the face of mistakes or incorrect executions.

Interpreting Fregean senses as procedures has already met with considerable criticisms. Dual aspect theorists like Putnam 1975, have criticized the idea of sense as a "procedure for determining reference". Evans, too (1982, note 17), has attacked the procedural notion of sense held by Dummett, interpreted as an ideal method of *verification* of the truth value of a sentence, given the procedures of the individual subsentential parts. However, in these criticisms it is assumed that the referent of a sentence is its truth value; but, as is often remarked, the argument given by Frege to justify truth values as the referents of a sentence is not compelling; if we pursue our strategy, it seems to me that we are bound to make a major revision of Frege's layers of semantic analysis; we have to abandon the idea that truth is the referent of a sentence, and draw a distinction analogous to the one made by Frege for predicates: reference and extension. Truth can therefore be considered the *extension* of a sentence, while we may treat the truth condition as taking the place given to concepts for predicates: the referent. Below, I have tried to give a schema, copying and changing at the same time the one sent by Frege to Husserl[30] :

	sentence	*predicate*	*singular term*
Cognitive sense	procedure	procedure	procedure
Semantic sense	truth condition	concept	constant function
Extension	truth value	class	individual

in the present framework, we may give a further clarification in defining levels of understanding; not only the number or the depth of the consequences of an assertion, but also the steps needed to build the assertion itself.

[29] See Chalmers 2006 § 3.4.

[30] See Frege, WB pp. 96-98.

In this schema the difference with the Fregean is the following: the distinction between reference and extension is applied not only to predicates, but also to singular terms and sentences. Using a singular term we refer to a constant function from possible worlds to individuals, and using a sentence we refer to a function from possible worlds to truth values. But, as with every function, we might think of procedures associated to the functions as an explicatum of the cognitive notion of sense (and we may have different procedures for different possible worlds).

8 Frege's requirements and Contemporary logic

What if Frege were discussing his conception of sense whilst having in mind developments in contemporary logic, especially with application to computability, like for instance implementation in automatic proof checking? He might look at his definition of sense given in *Grundgesetze* in a new vein. Saying that with our stipulations we determine in which conditions a sentence refers to the truth; therefore the sense of a sentence is "the thought that those conditions are satisfied" [Frege Gg 32] Traditionally most scholars have interpreted this sentence as if the thought were a representation of the truth conditions; but what Frege says can also be interpreted as if the thought were conceived as representing the steps which compute the truth conditions. Both interpretations are available and, with the idea of procedures attached to functions which are needed in any implementation of a logical system for it to work properly, we may save both the traditional idea of semantic sense or truth conditions as functions from possible worlds to truth values, and the idea of sense as different possible procedures attached to those functions.

The requirements on normal form suggested by Frege in his letter to Husserl can be interpreted in such a way: we may have a unique normal form for different expressions, signifying the standard truth conditional content (what Perry would call referential content). However, different expressions which can be used for different purposes, can be explicitly described as different procedures which give the same truth conditions.

Here we should enter a domain of different kinds of computational logics dealing with uncertainty – default logics, non monotomic logics, probabilistic logics, multi context theories – which Frege could not have imagined in his time. Certainly Frege was mainly interested in the eternal realm of mathematics, with his platonistic view; however he realized the importance of discussing the limited access to this ideal realm, and he might have been at least curious of contemporary discussions on computing machines, which share with humans the problem of limited resources in information and computational capacities. The attempts to objectively study these limitations and the procedures which are devised to control these capabilities are a development of a formalism Frege contributed to inventing, and of whose basic problem he was deeply aware. It is therefore not completely incongruent to see a Fregean attitude in contemporary treatments of formalizations of uncertain reasoning:
- distinguishing different procedures depending on the needs and goals of speakers, as Frege suggested with his distinction of a normal form and different expressions.
- enlarging the scope of logic from representing mathematical reasoning to also representing commonsense reasoning - as Frege suggested in the introduction of his *Begriffsschrift*.
- giving space to the cognitive role of differences of expressions without falling into psychologism
- taking care of the role of context of utterance, as Frege strongly suggested in his later writings, but also in the definition of his context principle.

These are very general remarks on a possible encounter of Fregean requirements and contemporary computational logic. Frege said, giving his *Nachlass* to his son: "It is not all gold, but there is gold in it". If we don't confine Frege to within the limits of his nineteenth century environment, but study his suggestions with today's eyes, we may find more gold than we had previously expected.

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